Basics of Matrices

Subject: Basic Mathematics II

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Dr. Mahesh Kumar Sharma
Assistant Professor
School of Basic and Applied Sciences

Matrices and Determinant

→ Matrices and determinants are very powerful mathematical tools used for solving problems that arise not only in mathematics but also in different fields of engineering, physics, economics, pharmaceutical science and statistics etc.

<u>Matrix</u> :- A matrix is an arrangement of numbers (real or complex) in rows (horizontal lines) and columns (vertical lines).

Example :-

$$\left.\underbrace{\begin{pmatrix} 2 & 3 & -4 \\ 1 & -2 & 6 \end{pmatrix}}_{3}\right]_{2}$$ is a matrix.

It has 2 rows and 3 columns.

$$\begin{array}{cccc} & & & \\ \multicolumn{3}{c}{\begin{pmatrix} 2 & 3 & -4 \\ 1 & -2 & 6 \end{pmatrix}} & \begin{array}{l} \longrightarrow I^{st} \text{ Row} \\ \longrightarrow II^{nd} \text{ Row} \end{array} \\ \downarrow & \downarrow & \downarrow & \\ 1^{st} & 2^{nd} & 3^{rd} & \\ \text{col.} & \text{col.} & \text{col.} & \end{array}$$

The matrix can also be written as

$$\begin{bmatrix} 2 & 3 & -4 \\ 1 & -2 & 6 \end{bmatrix}$$

→ A matrix of order $m \times n$ is an arrangement of mn numbers (elements) in m rows and n columns.

$$
\begin{array}{cc}
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\
a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn}
\end{bmatrix}
&
\begin{array}{l}
- \text{I}^{st} \text{ Row} \\
- \text{II}^{nd} \text{ Row} \\
\\
\\
\\
- m^{th} \text{ Row}
\end{array}
\\
\begin{array}{ccccc}
1^{st} \text{ col.} & 2^{nd} \text{ col.} & 3^{rd} \text{ col.} & & n^{th} \text{ col.}
\end{array}
&
\end{array}
$$

is a matrix of order $m \times n$.

$$\rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$ is a 3×3 matrix

$$\rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3}$$ is a 2×3 matrix.

row $\times$ column

Types of matrices

I) Row matrix OR Row vector

A matrix having only one Row is called a Row matrix or a Row vector.

Ex.) $(a_{11} \quad a_{12} \quad a_{13})$ is a row matrix of order 1×3

Ex) $(3 \quad 4 \quad 5)$ is a row matrix of order 1×3

II) Column matrix or column vector

A matrix having only one column is called a column matrix or a column vector.

Ex.) $\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix}$ is a column matrix of order 3×1

Ex.) $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ is a column matrix of order 2×1

III) Null matrix or Zero matrix

A matrix in which all the elements are zeros is called a null matrix or zero matrix (void matrix).

Ex) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is a zero matrix of order 2×3.

(VI) Square matrix

A matrix in which the number of rows is equal to the number of columns is called a square matrix. otherwise, it is called a rectangular matrix or simply a matrix.

Ex.) $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ is a square matrix of order 2×2 or Order 2.

Ex.) $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 4 & 6 \\ 0 & 5 & -5 \end{bmatrix}$ is a square matrix of order 3×3 or order 3.

(V). <u>Diagonal matrix</u> : A diagonal matrix is a square matrix in which all elements except the elements in the principal diagonal are zeros.

<u>Ex.</u>) $\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$ and $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ are diagonal matrices of order 2 and 3 respectively.

(VI) Scalar matrix :- It is a diagonal matrix in which all the elements in the principal diagonal are same.

Ex) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ are scalar matrices of order 3 and 2 respectively.

(VII) Unit matrix or Identity matrix

A unit matrix is a square matrix in which every element in the principal diagonal is 1 and all other elements are zero.

ex.) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is identity matrix of order '3'.

Operation on Matrices

(6)

1.1 Scalar multplication

A matrix can be multiplied by a scalar (a constant).

If A is any matrix and k is any scalar, then kA is a matrix obtained by multiplying every element of A by k.

i.e. if $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$; then

$$kA = \begin{bmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \end{bmatrix}.$$

For Ex.) If $A = \begin{bmatrix} 3 & 4 & 2 \\ 2 & -5 & 6 \end{bmatrix}$, Then

(i) $2A = 2\begin{bmatrix} 3 & 4 & 2 \\ 2 & -5 & 6 \end{bmatrix} = \begin{bmatrix} 6 & 8 & 4 \\ 4 & -10 & 12 \end{bmatrix}$

(ii) $-3A = -3\begin{bmatrix} 3 & 4 & 2 \\ 2 & -5 & 6 \end{bmatrix} = \begin{bmatrix} -9 & -12 & -6 \\ -6 & 15 & -18 \end{bmatrix}$

(iii) $\frac{1}{2}A = \frac{1}{2}\begin{bmatrix} 3 & 4 & 2 \\ 2 & -5 & 6 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & 2 & 1 \\ 1 & -\frac{5}{2} & 3 \end{bmatrix}$

2.) Addition of matrices (7)

Two matrices can be added or substracted only if their orders are same.

If A and B are matrices of order $m \times n$, then $A + B$ is of same order and it is defined as the matrix obtained by adding the corresponding elements of A and B.

i.e. if $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$

Then $A+B = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$

$$= \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \end{bmatrix}$$

→ For Example if $A = \begin{bmatrix} 3 & 4 & 2 \\ 5 & -2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 0 \\ -3 & 2 & 4 \end{bmatrix}$

Then, $A+B = \begin{bmatrix} 3 & 4 & 2 \\ 5 & -2 & 6 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 0 \\ -3 & 2 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 3+2 & 4-1 & 2+0 \\ 5-3 & -2+2 & 6+4 \end{bmatrix} = \begin{bmatrix} 5 & 3 & 2 \\ 2 & 0 & 10 \end{bmatrix}$$

Ex.) If $\begin{bmatrix} 2 & 3 \\ 7 & 5 \end{bmatrix} + \begin{bmatrix} 2 & x-2 \\ y-1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 7 & 10 \end{bmatrix}$;

then find x and y.

Sol.)

we have $\begin{bmatrix} 2 & 3 \\ 7 & 5 \end{bmatrix} + \begin{bmatrix} 2 & x-2 \\ y-1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 7 & 10 \end{bmatrix}$

$$\Longrightarrow \begin{bmatrix} 2+2 & 3+x-2 \\ 7+y-1 & 5+5 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 7 & 10 \end{bmatrix}$$

$\therefore\ x+1=1$ and $y+6=7$

$\Rightarrow\ x=0$ and $y=7-6=1$

Hence; $x=0$ and $y=1$